On the Possibility of Measuring Magnetic Fields by Scattered Light

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Abstract

In the following report we discuss the possibility of measuring magnetic fields by scattered light. In the case of uncorrelated electrons (i.e. $\alpha \ll 1$) the electron spectrum is modulated with the electron gyration frequency if a certain condition is fulfilled. The essential parameters are discussed and some spectra are computed numerically.

1. Introduction

The magnetic field in a plasma is a very important quantity and all methods of measuring it are, therefore, of great interest. Unfortunately, it is not at all as easy to measure plasma magnetic fields as one might at first believe. All methods used up to now have serious disadvantages.

The first and most direct method is to use magnetic probes, which allow local measurement of the magnetic field. The plasma may be strongly disturbed by the presence of probes, however, and so this method has had to be abandoned for measurements in hot plasmas, for instance, in fast theta pinches. This does not mean that probe measurements are of no use at all. They are still important and useful for studying colder plasmas, preionization phenomena and early phases of pinches, etc.

Another possibility is offered by the Zeeman effect. In this case one measures the line shift produced by the magnetic field. Here it has to be assumed that Zeeman shifted lines exist with sufficient intensity. In a pure and hot deuterium plasma, for example, this is not the case. To achieve sufficient intensity one has to add high Z-impurities to such an extent that the plasma properties are essentially changed and the measurement may become useless.

A third method makes use of the Faraday effect. It does not disturbe the plasma as do the above mentioned methods. It has another disadvantage, however. One measures the integral $\int n(z)\, B(z)\, dz$ along the path of the plane polarized light beam parallel to the magnetic field $B(z)$ ($n(z)$ is the electron density), which produces a rotation of the plane of polarisation. Thus one gets an average value of nB but no information on local magnetic fields in general.

In principle at least, there is another method which uses the scattering of light in a magnetoplasma. It is based on the fact that the scattered spectrum shows the influence of magnetic fields under certain conditions, i.e. it may show a modulation with both the gyrofrequencies of electrons and ions [1,2,3] . In the case of uncorrelated electrons, i.e. for $\alpha = (k\lambda_D)^{-1} \ll 1$, (where λ_D is the Debye length and k the modulus of the scattering vector) one is left with the electron spectrum only, which may be modulated with the electron gyrofrequency. This method is most promising as far as measurements of magnetic fields are concerned, and in the following attention will therefore be restricted to this case.

2. The electron spectrum in the presence of a magnetic field.

In this section we shall derive an expression for the form of the scattered spectrum, which has been given in more detail in the above-mentioned papers [1,2,3] . The intensity is given by the usual Thomson scattering cross section of the electron and will not be discussed. As is well known, one has to use laser light for reasons of intensity. This report deals only with the form of the spectrum and with the prospects of measuring the modulation produced by the magnetic field.

Let us consider an electron gyrating in a magnetic field having perpendicular velocity $v_\perp$ and moving parallel to the field with parallel velocity $v_\parallel$. Light incident with the wave vector $\vec{k}_o$ and scattered with $\vec{k}_s$ is shifted from its initial frequency ω_o to $\omega_o + \Delta\omega$. Neglecting the quadratic Doppler effect we get

$$(1) \qquad \Delta\omega = (\vec{k}_s - \vec{k}_o) \cdot \vec{v} = \vec{k} \cdot \vec{v}$$

where $\vec{k}$ is the scattering vector,

$$(2) \qquad \vec{k} = \vec{k}_s - \vec{k}_0 \quad ; \quad k = |\vec{k}| = \frac{4\pi}{\lambda_0} \sin\frac{\theta}{2}$$

where λ is the wavelength of the incident light and Θ the scattering angle (Fig. 1). It is assumed that the laser pulse is of sufficiently long duration, i.e. long compared with any other typical time occurring in the problem. Furthermore it is assumed that the light frequency $\omega_0 = k_0 c$ is large compared with the gyration frequency ω_g, which, in turn, is assumed to be large compared with the collision frequency of the electrons. We can then proceed as follows.

We split $\Delta\omega$ into a perpendicular part,

$$(3) \qquad \Delta\omega_\perp = \vec{k} \cdot \vec{v}_\perp$$

and a parallel part

$$(4) \qquad \Delta\omega_\parallel = \vec{k} \cdot \vec{v}_\parallel = k v_\parallel \sin\beta$$

The parallel part is constant for a given electron with constant parallel velocity $v_\parallel$, i.e. as long as it does not collide. The perpendicular part, however, oscillates with the gyration frequency, i.e. the scattered light may be described as frequency modulated. β being the angle between $\vec{k}$ and the perpendicular to the magnetic field (Fig. 1), equation (3) leads to

$$(5) \qquad \Delta\omega_\perp = k \cos\beta \, v_\perp \sin(\varphi_0 - \omega_g t)$$

where ω_g is the gyration frequency

$$(6) \qquad \omega_g = \left| \frac{eB}{m} \right|$$

and φ_0 an initial phase which will be taken as zero in the following, this amounting to a special choice for the origin of time. Thus the scattered light has the frequency (angular velocity)

(7) $$\omega = \omega_0 + kv_{\parallel} \sin\beta - kv_{\perp} \cos\beta \sin\omega_g t$$

and the phase

(8) $$\Phi = \int_0^t \omega(t')\,dt' = (\omega_0 + kv_{\parallel}\sin\beta)t + \frac{kv_{\perp}\cos\beta}{\omega_g}\cos\omega_g t$$

The scattered wave is then of the form

(9) $$W \sim \cos\left[(\omega_0 + kv_{\parallel}\sin\beta)t + \frac{kv_{\perp}\cos\beta}{\omega_g}\cos\omega_g t\right]$$

and its Fourier analysed form is

(10) $$W \sim \sum_{n=-\infty}^{+\infty} J_n\left(\frac{kv_{\perp}\cos\beta}{\omega_g}\right)\cos\left[(\omega_0 + kv_{\parallel}\sin\beta)t + n\omega_g t + \frac{n\pi}{2}\right]$$

One observes discrete lines at the frequencies $\omega_0 + kv_{\parallel}\sin\beta \pm n\omega_g$ Their amplitude is given by $J_n\left(\frac{kv_{\perp}\cos\beta}{\omega_g}\right)$, J_n being the n^{th} Bessel function. The intensity is then J_n^2.

This is the case of a single electron. If one has many uncorrelated electrons one can simply add the intensities taking into account the velocity distribution of the electrons. Assuming a Maxwellian distribution one gets the intensity of the n^{th} line

(11) $$S_n \sim \int_0^\infty J_n^2\left(\frac{k\cos\beta}{\omega_g} v_\perp\right) \exp\left(-\frac{m v_\perp^2}{2KT}\right) v_\perp \, dv_\perp$$

$$\sim \int_0^\infty J_n^2\left(\frac{k\cos\beta}{\omega_g}\sqrt{\frac{2KT}{m}}\, z\right) \exp(-z^2)\, z \, dz$$

$$= \frac{1}{2} \exp\left(-\frac{k^2\cos^2\beta\, KT}{m\,\omega_g^2}\right) I_n\left(\frac{k^2\cos^2\beta\, KT}{m\,\omega_g^2}\right)$$

The latter integral can be found in Watson's treatise [4]. I_n is the modified Bessel function. Putting

(12) $$a = \frac{k^2\cos^2\beta \;\; KT}{m\,\omega_g^2}$$

and normalising so that the total intensity in all lines is unity we get

(13) $$S_n = \exp(-a)\, I_n(a) = \int_{-\infty}^{+\infty} s_n(\omega)\, d\omega$$

because

(14) $$\sum_{n=-\infty}^{+\infty} \exp(-a)\, I_n(a) = 1$$

Due to the spread of $v_\parallel$ each line is Doppler broadened, i.e. each line has Gaussian profile of the form

(15) $$\exp\left(-\frac{m v_\parallel^2}{2KT}\right) = \exp\left(-\frac{m(\Delta\omega_\parallel)^2}{k^2\sin^2\beta\; 2KT}\right)$$

Putting

(16) $$\Omega = \frac{\Delta\omega}{\omega_g} = \frac{\omega-\omega_0}{\omega_g}$$

a single line is described by

(17) $$s_n(\Omega) = \frac{\exp(-a)}{\sqrt{\pi b}} I_n(a) \exp\left[-\frac{(\Omega-n)^2}{b}\right]$$

where

(18) $$b = \frac{k^2 \sin^2\beta \, 2KT}{m \omega_g^2} = 2a \, tg^2\beta$$

and the total spectrum is

(19) $$S(\Omega) = \sum_{n=-\infty}^{+\infty} s_n(\Omega) = \frac{\exp(-a)}{\sqrt{\pi b}} \sum_{n=-\infty}^{+\infty} I_n(a) \exp\left[-\frac{(\Omega-n)^2}{b}\right]$$

with

(2o) $$\int_{-\infty}^{+\infty} s(\Omega) \, d\Omega = S = 1$$

The form of the spectrum depends on the two parameters a and b. Roughly speaking a is responsible for the form of the spectrum as a whole, i.e. for its envelope, while b is responsible for the degree of modulation.

One may distinguish two limiting cases:

a) $a \ll 1$: In this case the spectrum practically contains its central line only because the intensity of the other lines becomes negligibly small. The width is given by $\sqrt{b}$ and can be compared with the width of the spectrum in the absence of a magnetic field. It appears that the magnetic field contracts the spectrum by a factor $\sin\beta$. a is small either if the magnetic field is very large of if $\beta \approx \frac{\pi}{2}$. In the latter case $\sin\beta \approx 1$ and the spectrum is the same as in the absence of the magnetic field.

b) $a \gg 1$. If in this case $b \ll 1$ the lines to not overlap and the envelope of the different sharp linies is given by $I_n(a)$ considered as a function of n. Using asymptotic formulas for $I_n(a)$ one can easily show that the envelope is very close to a Gaussian profile with the width $\sqrt{2a}$. From $a \gg 1$, $b \ll 1$ it follows that $\cos\beta \approx 1$, i.e. the envelope corresponds to the spectrum without a magnetic field.

Case (b) might be useful for the measurement of magnetic field and we shall give a detailed discussion of the parameters involved and some numerically obtained spectra in the following section. Before that we conclude this section with some more general remarks.

An electron gyrating in a magnetic field is somewhat like an atom, the Coulomb forces being replaced by Lorentz forces. Its spectrum consists of the gyrofrequency and its harmonics. The scattering causes a shift of this spectrum to a higher frequency range and thus is similar to Raman scattering.

In a dense plasma the radiation of a gyrating electron is absorbed and cannot be observed from outside the plasma. As a result of scattering its spectrum can be shifted to an observable range of frequencies if the light frequency ω_0 is sufficiently large, i.e. well above all resonances of the plasma under investigation.

Shifting the spectrum to higher frequencies has another important consequence concerning the intensities. A nonrelativistic dipole or a nonrelativistic gyrating electron emits the bulk of its intensity into the fundamental line and the intensities of the harmonics are very small [5,6] . Consider the simplest case of a linear harmonic dipole. The amplitude of the n^{th} harmonic is essentially given by

$$J_n\left(\frac{nv}{c}\cos\psi\right) = J_n\left(\frac{2\pi\ell}{\lambda_n}\cos\psi\right)$$

where ψ is the angle between the dipole and the direction of observation and where ℓ is the length of the dipole and λ_n the wavelength of the n^{th} harmonic. For small argument $n v/c \sim \ell/\lambda_n$, i.e. for the nonrelativistic dipole,

$$I_n \sim \left(\frac{\ell}{\lambda_n}\right)^n \sim \left(\frac{v}{c}\right)^n$$

The physical reason for the appearance of harmonics even for a harmonic dipole is that the light observed at a given point comes from different positions occupied by the oscillating electron at different times. So the observed phase is still periodic but not harmonic. From this point of view it is plausible that with $\ell/\lambda_n \ll 1$ the higher harmonics are not important because the phase differences are then very small. For a gyrating electron which is a two-dimensional dipole the reasoning is very similar. One has to replace ℓ by twice the radius of gyration 2R. For the shifted spectrum the situation has completely changed, however, we have to expect that the essential quantity now is R/λ_o. This is indeed so. Consider the argument of I_n in equation (9), which is

$$\frac{k v_\perp}{\omega_g} \cos\beta = kR\cos\beta = \frac{R}{\lambda_o} 4\pi \sin\frac{\theta}{2} \cos\beta$$

and this is no longer a small quantity because $\lambda_o \ll \lambda_n$. This explains why we can have the harmonics in the shifted spectrum with appreciable intensity though they do practically not appear in the original one.

Let us mention a problem which bears some resemblance to the one discussed above. Using microwave techniques Landauer [7] has observed harmonics of the gyrofrequency emitted by a cold PIG discharge in helium. The electrons are certainly not relativistic and the question arises how the emission of harmonics is possible. According to Landauer the explanation is that the plasma has a very high index of refraction in this range of frequencies which provides another way of achieving larger values of R/λ. This means that the velocity of the electrons is comparable with the velocity of light in the plasma , which is much smaller than the velocity of light in vacuo. For this reason Landauer has introduced the notion of "quasirelativistic" electrons [7].

The results of the present section can also be applied to anisotropic velocity distributions of the electrons with different parallel and perpendicular temperature ($T_\perp \neq T_\parallel$). T has to be replaced by $T_\parallel$ in equation (18) for b and by $T_\perp$ in equation (12) for a. The relation (18) between a and b is no longer valid, however.

3. Discussion of the relevant parameters

In this section we need the equations (12), (18) und (19) of the preceding section only. It is obvious that magnetic fields can be measured for small values of b only, $b \ll 1$. This means that the gyration frequency has to be sufficiently large compared with the shift produced by the mean parallel velocity. Thus we have

(21) $$\sqrt{b} = 0{,}395 \, \frac{\sin\frac{\theta}{2} \sin\beta \sqrt{T}}{B \lambda_0} \ll 1$$

At the same time we have to require

(22) $$\alpha^{-1} = k \lambda_D = 86{,}5 \, \frac{\sin\frac{\theta}{2}}{\lambda_0} \sqrt{\frac{T}{n}} \gg 1$$

A value of $\alpha^{-1} = 2$ is already sufficient, i.e. we assume

(23) $$\frac{\sin\frac{\theta}{2} \sqrt{T}}{\lambda_0} = \frac{\sqrt{n}}{43{,}25} \qquad \left(\alpha = \frac{1}{2}\right)$$

so that equation (21) yields

$$(24) \qquad \frac{\sin\beta\sqrt{n}}{B} \ll 110 \qquad \left(\alpha = \tfrac{1}{2}\right)$$

This condition is fulfilled, for example, for $\beta \approx$ a few $1o^{-2}$, $n \approx 1o^{16}$ cm^{-3} and $B \approx 1o^{5}$ G. If $\lambda_o = 7ooo$ Å $= 7 \times 1o^{-5}$ cm and $T \approx 1o^{6}$ ^oK, equation (23) gives $\sin\frac{\theta}{2} = o{,}162$, $\theta \approx 19^o$. The separation between the lines is $\omega_g = 1.76 \times 1o^{12}$ sec^{-1}, which is equivalent to $\Delta\lambda = 3.4$Å . The collision frequency is about $5 \times 1o^{8}$ sec^{-1}, i.e. much smaller than the gyrofrequency, in agreement with our initial assumption. So one can conclude that magnetic fields of about $1o^{5}$G in plasmas with electron densities of about $1o^{16}$ cm^{-3} can be measured in principle by light scattering.

We have numerically computed some spectra for different values of a and b. Examples are shown in Figs. 2 - 5. For better comparison they are normalized to unity at the center. The dashed curves represent the Gaussian profiles for the case of no magnetic field.

4. Conclusion

We have discussed the parameters which are of importance for measuring the magnetic fields in plasmas by light scattering. It is not the purpose of this report to discuss the perhaps formidable difficulties which may arise in possible experiments. We merely wish to draw the attention of experimentalists to this problem, which could perhaps lead to a very valueable diagnostic method.

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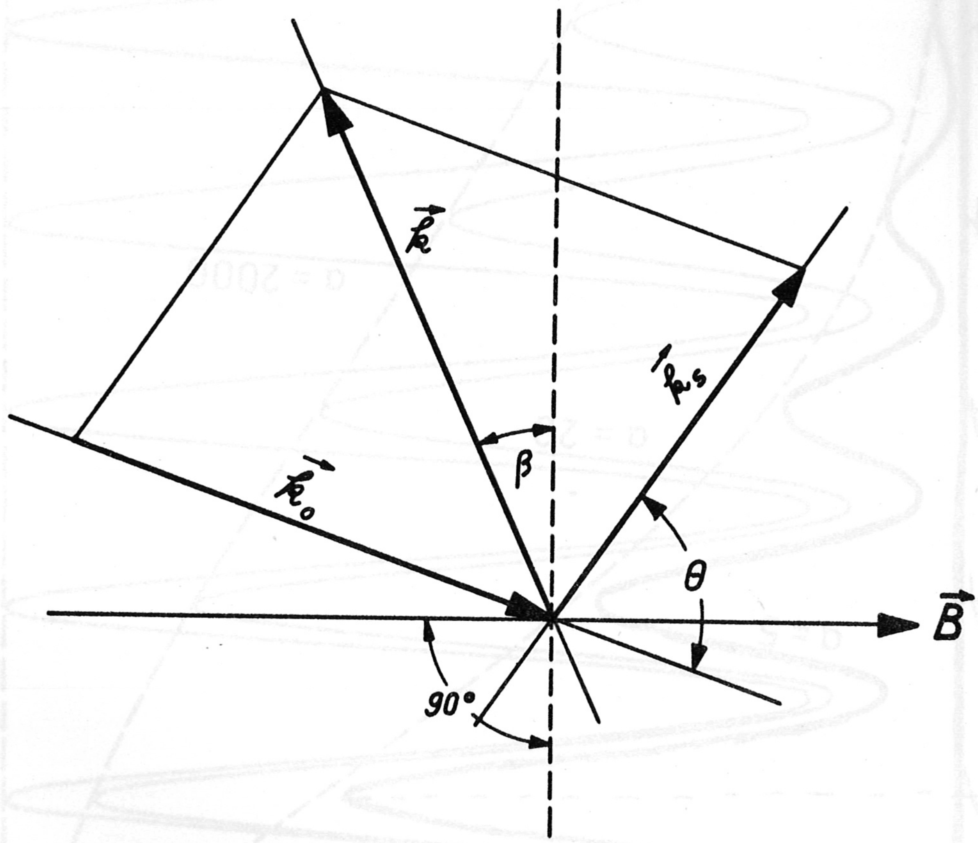
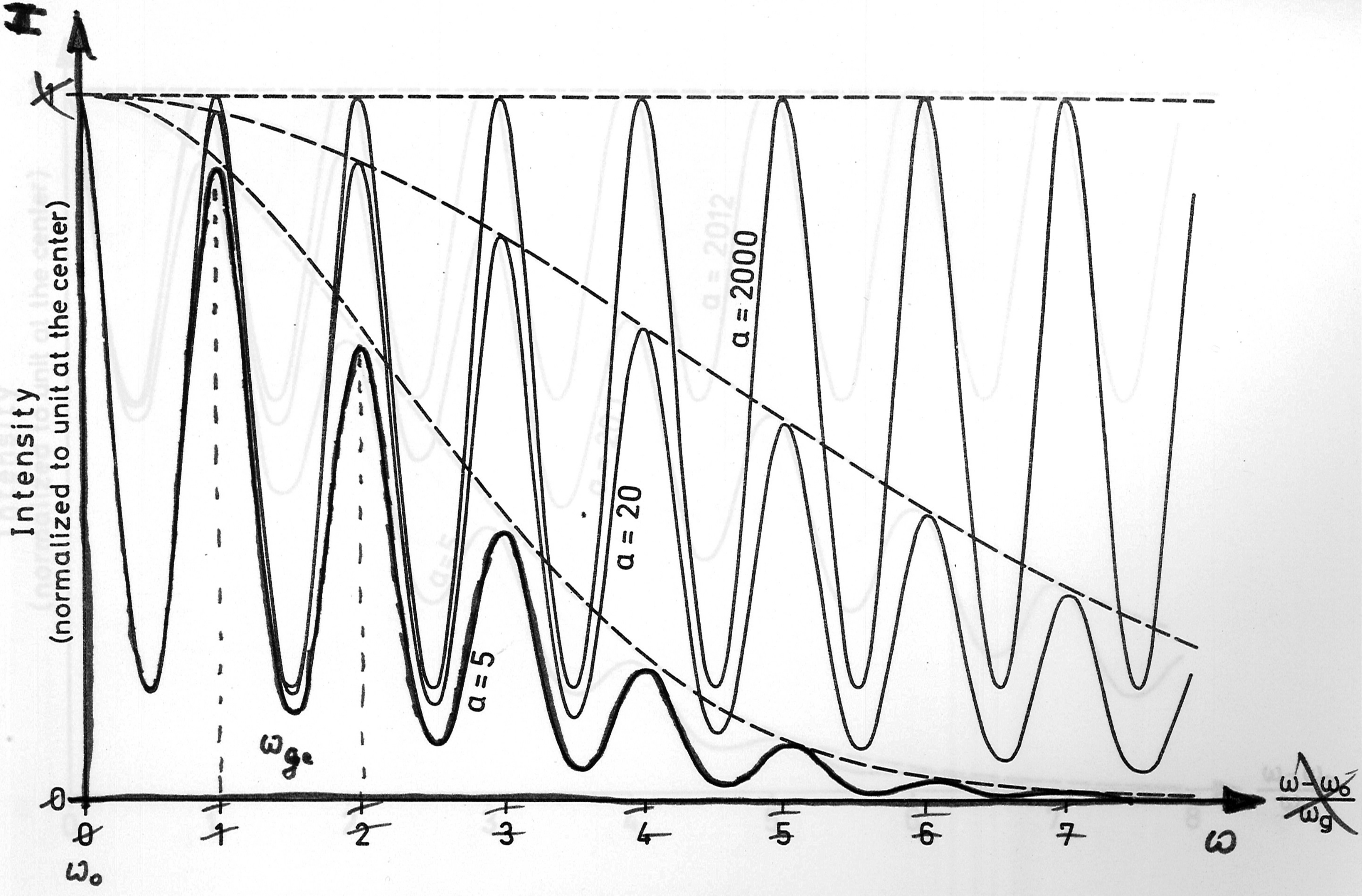


Fig. 1: Geometry of Scattering.



ω_{ge}

Fig. 2: Scattered Spectra for b = 0.1 and
a = 2000; 20; 5 .

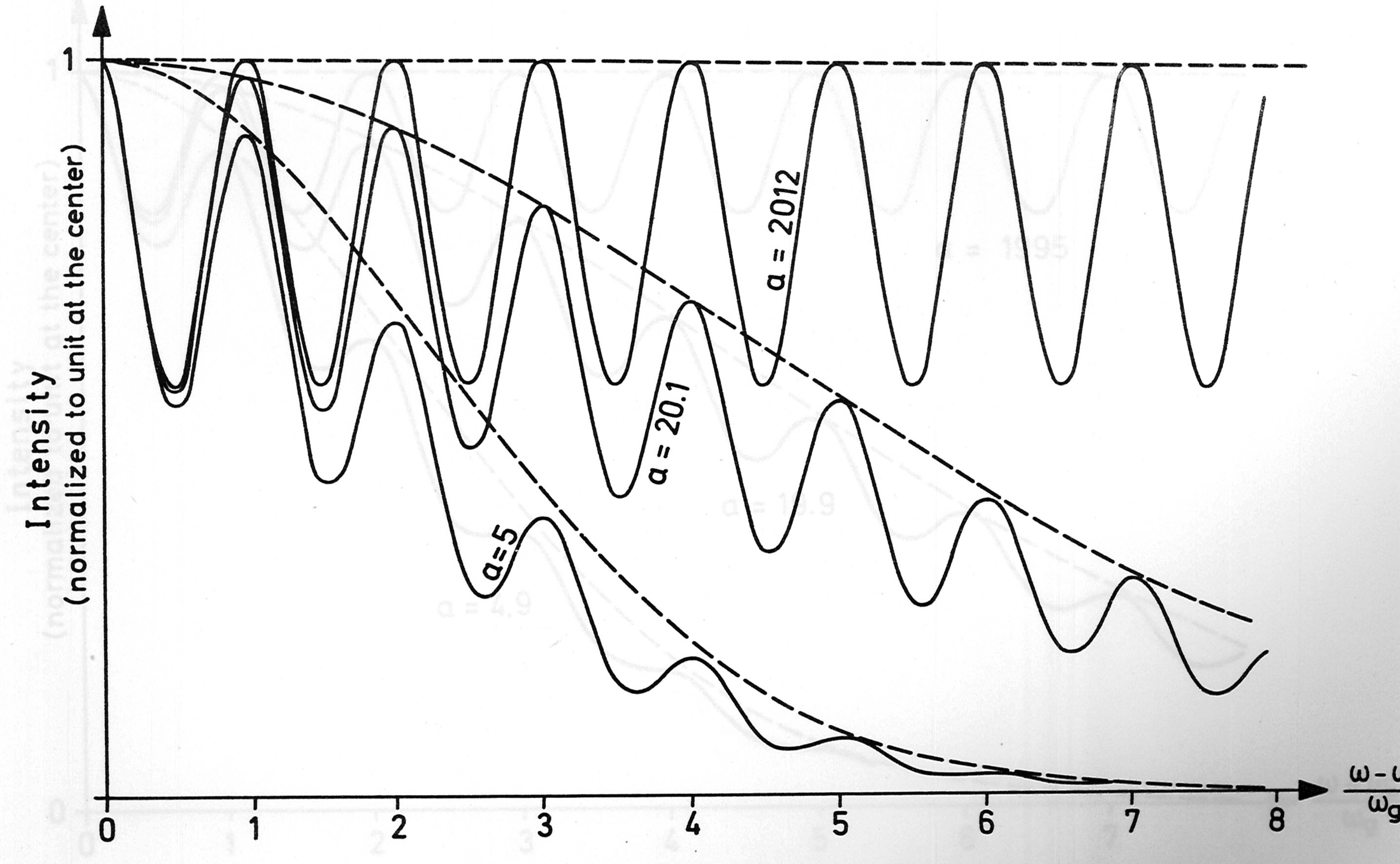
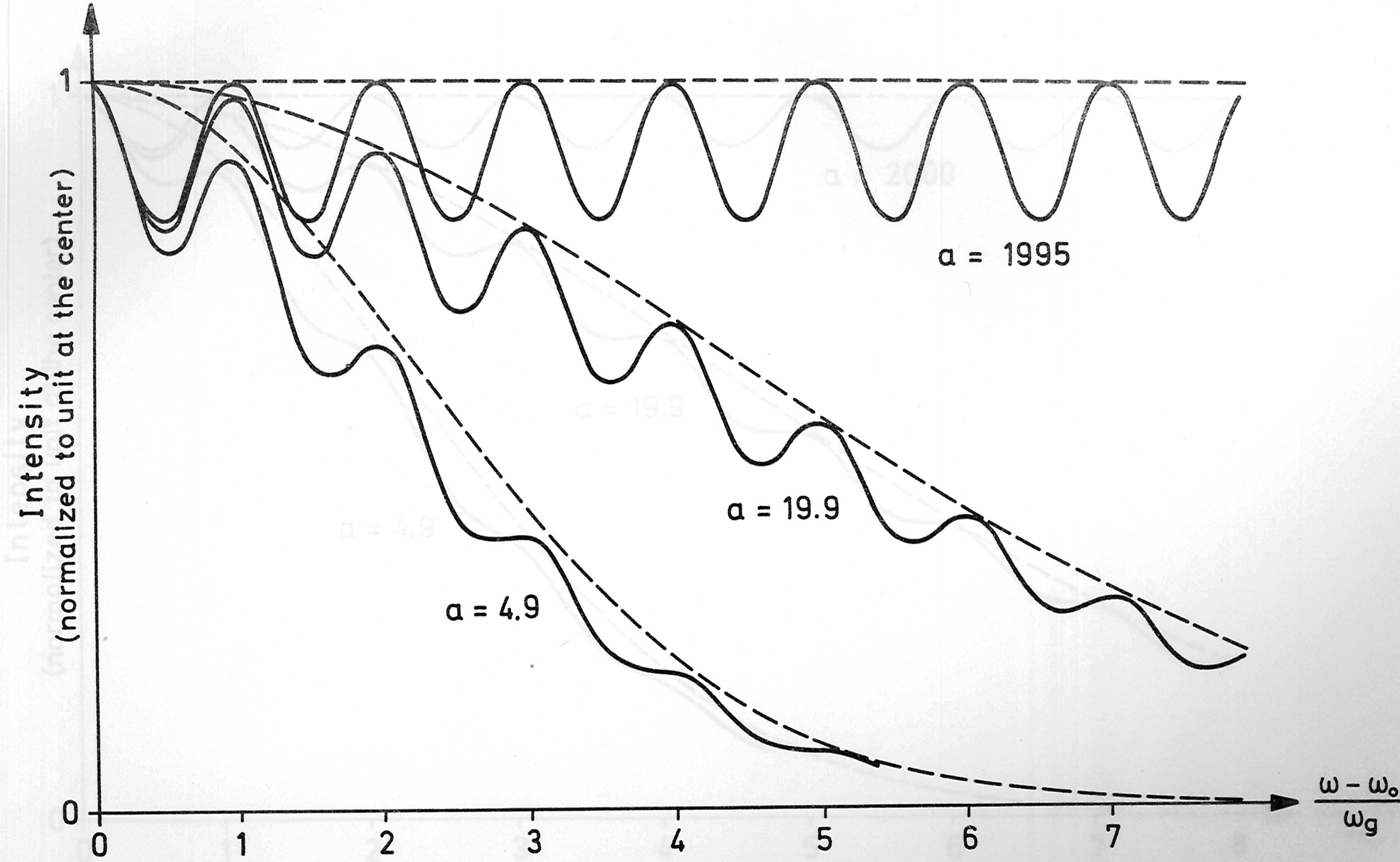


Fig. 3: Scattered Spectra for b = 0.2 and
a = 2012; 20.1; 5 .



<u>Fig. 4:</u> Scattered Spectra for b = 0.3 and
a = 1995; 19.9; 4.9 .

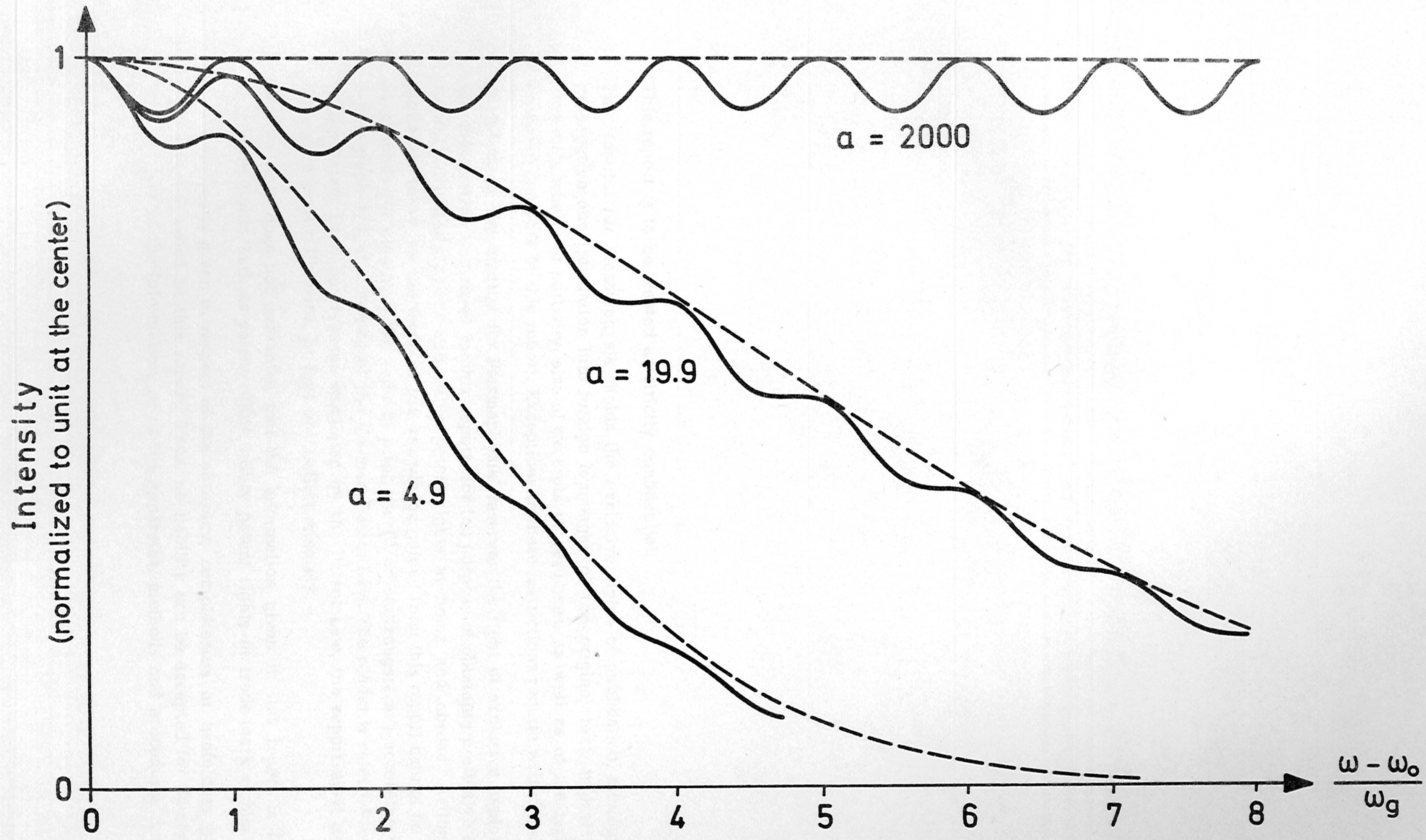


Fig. 5: Scattered Spectra for b = 0.4 and a = 2000; 19.9; 4.9 .